$\mathcal{N}$=4 Super Yang-Mills Low-Energy Effective Action at Three and Four Loops

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Abstract

We investigate the low-energy effective action in N=4 super Yang-Mills theory with gauge group $SU(n)$ spontaneously broken down to its maximal torus. Using harmonic superspace technique we prove an absence of any three- and four-loop corrections to non-holomorphic effective potential depending on $N=2$ superfield strengths. A mechanism responsible for vanishing arbitrary loop corrections to low-energy effective action is discussed.

Supersymmetry imposes the significant restrictions on a structure of effective action in field models. It is naturally to expect that the most strong restrictions have to arise in maximally extended rigid supersymmetric model, that is in $N=4$ super-Yang-Mills theory.

Recently Dine and Seiberg found that a dependence of $N=4$ supersymmetric Yang-Mills low-energy effective action on $N=2$ superfield strengths $\mathcal{W}$ and $\bar{\mathcal{W}}$ is exactly fixed only by general properties of the quantum theory under consideration like finiteness and scale independence [1]. According to ref [1] the leading low-energy contributions to effective action in $N=4$ SYM with gauge group $SU(2)$ spontaneously broken down to $U(1)$ are described by non-holomorphic effective potential $\mathcal{H}(\mathcal{W},\bar{\mathcal{W}})$ of the form

$$\mathcal{H}(\mathcal{W},\bar{\mathcal{W}}) = c\log(\frac{\mathcal{W}^2}{\Lambda^2})\log(\frac{\bar{\mathcal{W}}^2}{\Lambda^2}) \tag{1}$$

Here Λ is some scale. The effective potential (1) possesses by two remarkable properties. First, the corresponding effective action

$$\int d^4x d^8\theta \mathcal{H}(\mathcal{W},\bar{\mathcal{W}}) \tag{2}$$

is scale independent. Second, any quantum corrections, if they exist at all, are included into a single constant c.

The explicit calculations of the non-holomorphic effective potential and finding the coefficient c in one-loop approximation have been carried out in refs [2-4]. Extension of the above results for $N = 4$ Yang-Mills theory with gauge group $SU(n)$, $n > 2$, spontaneously broken down to its maximal torus have been developed in refs [5-8]. General structure of low-energy effective action in $N = 2, 4$ superconformal invariant field models was investigated in ref [9].

In the paper [1] Dine and Seiberg presented the qualitative arguments based on principle of naturalness [10] (see application of this principle to SUSY theories in ref [11]) that effective potential (1) gets neither perturbative nor non-perturbative quantum corrections beyond one loop. Therefore the expression (1) together with the results of one-loop calculations of the coefficient c [2-4] determines exact low-energy effective action in $N = 4$ $SU(2)$ Yang-Mills theory. The above arguments have also been discussed in refs [8,15]. Another approach leading to the same conclusion about structure of low-energy effective action was developed in recent papers [22].

We would like to pay an attention that a mechanism providing an absence of higher loop corrections to non-holomorphic effective potential in $N = 4$ Yang-Mills theory is unknown up to now. The firm results concern only two-loop approximation where the corresponding corrections are prohibited by $N = 2$ supersymmetry [12] (see also direct two-loop calculations in refs [13,14]).

An interesting aspect of $N = 4$ Yang-Mills theory with gauge group $SU(n)$, $n > 2$, has been recently pointed out in refs [8,15]. The symmetry arguments do not prohibit an appearance of some new invariant structures, besides logarithmic, in non-holomorphic effective potential which are absent at $n = 2$. The direct calculations [5-8] do not confirm such structures in one-loop approximation. However a question concerning their appearance at higher loops is open.

In this paper we are going to develop a technique for investigating a structure of the non-holomorphic effective potential at higher loops, to find a mechanism providing a cancellation of higher-loop contributions, and to clarify situation concerning the non-logarithimic corrections to low-energy effective action in $N = 4$ Yang-Mills theory with gauge group $SU(n)$, $n > 2$, spontaneously broken to its maximal torus. To be more precise, we investigate a structure of three- and four-loop supergraphs and show how $N = 4$ supersymmetry provides an efficient mechanism of supergraph cancellations.

We consider $N = 4$ Yang-Mills theory formulated in terms of $N = 2$ superfields and get $N = 2$ Yang-Mills theory coupled to hypermultiplet in adjoint representation. The most convenient and simple way to carry out quantum calculations in $N = 2$ SUSY models is given by harmonic superspace approach [16-18] which is used in the paper. The various implementations of this approach to effective action in $N = 2$ SUSY theories are discussed in refs [3,7,12,19,20].

The starting point of our consideration is the classical action of the $N = 4$ super-Yang-Mills theory in q-hypermultiplet realization written in harmonic superspace [16,17]

$$S = \frac{1}{g^2}\mathrm{tr}\int d^{12}z\sum_{n=2}^{\infty}\frac{(-\mathrm{i})^n}{n}\int du_1\cdots du_n\frac{V^{++}(z,u_1)\cdots V^{++}(z,u_n)}{(u_1^+u_2^+)(u_2^+u_3^+)\cdots(u_n^+u_1^+)} +$$

$$+ \quad \int d\zeta^{(-4)} du \breve{q}^{+}(D^{++} + iV^{++})q^{+}. \tag{3}$$

The denotions introduced in the paper [12] are employed here and further.

The calculations are carried out in framework of $N = 2$ background field method [19]. We make background-quantum splitting by the rule

$$V^{++} \to V^{++} + g\, v^{++} \tag{4}$$

and construct the corresponding Faddeev-Popov ghost action in the form [12]

$$S_{gh} = \mathrm{tr} \int d\zeta^{(-4)} du \mathbf{b} (\nabla^{++})^2 \mathbf{c} - \mathrm{i}\, g\, \mathrm{tr} \int du\, d\zeta^{(-4)}\, \nabla^{++} \mathbf{b}\, [v^{++}, \mathbf{c}]\, . \tag{5}$$

The background-dependent superpropagators in the theory with action of $N = 2$ gauge multiplet and $N = 2$ matter hypermultiplet (3) and action of ghosts (5) have been obtained in [12] and look like

$$\begin{aligned}
< v_\tau^{++}(1)\, v_\tau^{++}(2) > \;&=\; -\frac{\mathrm{i}}{\widehat{\Box}} (\overrightarrow{\mathcal{D}_1^+})^4 \Big\{ \delta^{12}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2) \Big\} \\
< q_\tau^{+}(1)\, \breve{q}_\tau^{+}(2) > \;&=\; \frac{\mathrm{i}}{\widehat{\Box}} (\overrightarrow{\mathcal{D}_1^+})^4 \left\{ \delta^{12}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \right\} (\overleftarrow{\mathcal{D}_2^+})^4 \\
< \omega_\tau(1)\; \omega_\tau^{\mathrm{T}}(2) > \;&=\; -\frac{\mathrm{i}}{\widehat{\Box}} (\overrightarrow{\mathcal{D}_1^+})^4 \left\{ \delta^{12}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right\} (\overleftarrow{\mathcal{D}_2^+})^4 \\
< \mathbf{c}_\tau(1)\; \mathbf{b}_\tau(2) > \;&=\; -\frac{\mathrm{i}}{\widehat{\Box}} (\overrightarrow{\mathcal{D}_1^+})^4 \left\{ \delta^{12}(z_1 - z_2) \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} \right\} (\overleftarrow{\mathcal{D}_2^+})^4\, .
\end{aligned} \tag{6}$$

with the operator $\widehat{\Box}$ of the form [12]

$$\begin{aligned}
\widehat{\Box} \;=\;& \mathcal{D}^m \mathcal{D}_m + \frac{\mathrm{i}}{2} (\mathcal{D}^{+\alpha} \mathcal{W}) \mathcal{D}^-_\alpha + \frac{\mathrm{i}}{2} (\bar{\mathcal{D}}^+_{\dot\alpha} \bar{\mathcal{W}}) \bar{\mathcal{D}}^{-\dot\alpha} - \frac{\mathrm{i}}{4} (\mathcal{D}^{+\alpha} \mathcal{D}^+_\alpha \mathcal{W}) D^{--} \\
& + \frac{\mathrm{i}}{8} [\mathcal{D}^{+\alpha}, \mathcal{D}^-_\alpha] \mathcal{W} + \frac{1}{2} \{ \bar{\mathcal{W}}, \mathcal{W} \}
\end{aligned} \tag{7}$$

Here index τ denotes that corresponding superfields taken in τ-frame [16] where covariant derivatives $\mathcal{D}^i_\alpha$, $\bar{\mathcal{D}}^i_{\dot\alpha}$ are introduced to be independent of harmonic coordinates. The propagators look like (6) just in this frame (see details in [12]).

Our aim consists in calculations of three- and four-loop contributions to non-holomorphic effective potential. We consider the case of the gauge group $SU(n)$ spontaneously broken down to its maximal Abelian subgroup. The corresponding background strength $\mathcal{W}$ is a diagonal matrix of the form

$$\mathcal{W} = diag(\mathcal{W}_1, \mathcal{W}_2 \ldots \mathcal{W}_n); \; \sum_{i=1}^{n} \mathcal{W}_i = 0. \tag{8}$$

Since non-holomorphic effective potential depends only on background superfield strengths but not on their derivatives we omit everywhere all terms including derivatives of $\mathcal{W}$, $\bar{\mathcal{W}}$ in (6,7). Hence the operator $\widehat{\Box}$ in propagators (6) looks like

$$\widehat{\Box} \;=\; \mathcal{D}^m \mathcal{D}_m + \mathcal{W} \bar{\mathcal{W}}, \tag{9}$$

or, in the manifest form,

$$\widehat{\Box} \;=\; \begin{pmatrix} \mathcal{D}^m\mathcal{D}_m + \mathcal{W}_1\bar{\mathcal{W}}_1 & 0 & \dots \\ 0 & \dots & \dots \\ 0 & \dots & \mathcal{D}^m\mathcal{D}_m + \mathcal{W}_n\bar{\mathcal{W}}_n \end{pmatrix}. \tag{10}$$

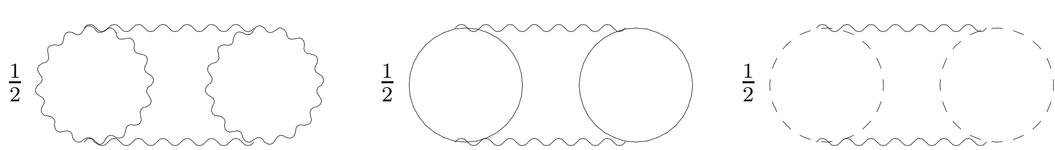
As a result we face a problem of calculating three- and four-loop supergraphs in the theory with constant background superfield strengths and operator $\widehat{\Box}$ given by (8) and (9) respectively.

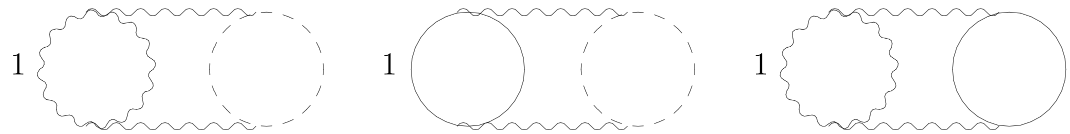
First of all, let us note that arbitrary L-loop supergraph provides non-zero contribution to non-holomorphic effective potential if and only if number of D-factors contained in it is equal to $8L$ or greater since contracting of any loop to a point by the rule $\delta^8(\theta_1-\theta_2)(D^+(u_1))^4(D^+(u_2))^4\delta^8(\theta_1-\theta_2) = (u_1^+u_2^+)^4\delta^8(\theta_1-\theta_2)$ requires 8 D-factors. Then, an arbitrary supergraph with P_v propagators of $N=2$ gauge superfield, P_m propagators of matter hypermultiplet, P_c propagators of ghosts, V_m vertices containing interaction with matter and V_c ones including interaction with ghosts contains the following number of D-factors:

$$N_D = 4P_v + 8P_m + 8P_c - 4V_m - 4V_c \tag{11}$$

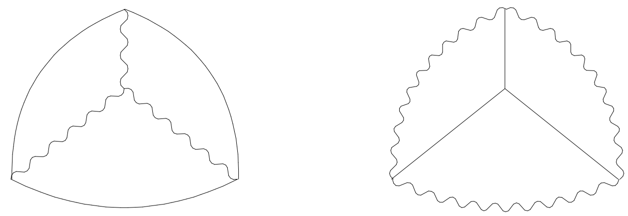
because of structure of propagators given in (6) with the operator $\widehat{\Box}$ has the form (9) and vertices corresponding to actions (3,5) (recall that transformation of vertex of the form $\int d\zeta^{(-4)}$ to an integral over whole superspace by the rule $\int d\zeta^{(-4)}(D^+)^4 = \int d^{12}z$ requires four D-factors). Then, it is easy to see that number of propagators of ghosts and matter is equal to number of vertices including interaction with ghosts and matter respectively since each pure ghost (matter) loop contains equal number of vertices and propagators, i.e. $P_m = V_m$, $P_c = V_c$. As a result, number of D-factors in arbitrary supergraph is equal to $N_D = 4P$ where P is a full number of propagators in corresponding supergraph. Hence any L-loop supergraph can contribute to non-holomorphic effective potential if and only if $P \geq 2L$. For example, two-loop supergraph should contain 4 and more propagators, and since number of propagators in two-loop supergraphs is no more than three we see that there is no two-loop contribution to $\mathcal{H}(\mathcal{W},\bar{\mathcal{W}})$ in accordance with conclusion of [12] where namely such an analysis was used to prove absence of two-loop non-holomorphic contribution. However a situation beyond two loops is much more complicated. In three- and four-loop supergraphs number of propagators should be no less than 6 or 8 respectively. Such supergraphs actually exist and are studied bellow.

Let us consider three-loop supergraphs corresponding to theory defined by actions (3,5). They are given by Figs. 1a, 1b. Here as usual wavy line is used for gauge propagator, solid line is for matter propagator, dashed line is for ghost propagator. The numbers near any supergraph at Fig. 1a will be explained later.



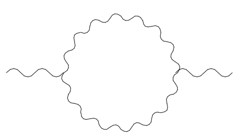


$Fig.1a$

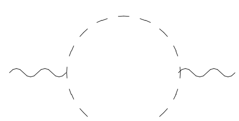


$Fig.1b$

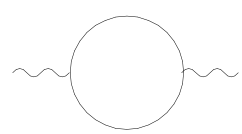
We are going to show that total contribution of the supergraphs given by Fig. 1a to non-holomorphic effective potential vanishes due to $N = 4$ supersymmetry. To clarify a mechanism providing manifestation of $N = 4$ supersymmetry in the supergraphs formulated in terms of $N = 2$ superfields we introduce a notion of $N = 4$ superpartner supergraphs. Three supergraphs are called $N = 4$ superpartners if they have the following structure. One of the supergraphs contains the gauge loop given by Fig. 2a and some system of the propagators associated with this loop. Another supergraph contains the matter loop given by Fig. 2c instead of gauge one and the same system of the propagators as in first case. Third supergraph contains the ghost loop given by Fig. 2b instead of gauge one and the same system of the propagators as in first case. The examples of systems of superpartner supergraphs are given also by Figs. 3a – 3c, Figs. 4a – 4c, Fig. 5. Appearance of such a set of supergraphs turned out to be typical for higher loop contributions. The simplest set of $N = 4$ superpartner supergraphs arising at one-loop order is given by Fig. 2a – Fig. 2c. We show that sum of three these supergraphs for background dependent superpropagators is equal to zero in the case of constant background superfield strengths.



$Fig.2a$



$Fig.2b$



$Fig.2c$

Structure of the supergraphs containing $N = 4$ superpartners in the case when propagators do not depend on background superfields was studied in details by GIKOS [18]. However we prove that the same result takes place when we consider non-holomorphic effective potential using the background field dependent propagators (7,10).

To evaluate contributions from supergraphs given by Figs. 2a – 2c one reminds that in τ-frame (where the propagators look like (6)) covariant harmonic derivatives ∇^{++}, ∇^{--} coincide with standard harmonic derivatives [16]. Then, the commutation relations $[\nabla^{++}, \mathcal{D}^+_A] = 0$ with $\mathcal{D}^+_A$ be either vector or spinor covariant derivative take place in any frame [12]. At the same time we pay attention to the fact that $[\mathcal{D}^+_\gamma, \mathcal{D}_{\alpha\dot\alpha}] = i\epsilon_{\gamma\alpha}\bar{\mathcal{D}}^+_{\dot\alpha}\bar{\mathcal{W}}$. Therefore one can

put $[\mathcal{D}^+_\gamma, \widehat{\Box}] = 0$ in the sector of non-holomorphic effective potential.

Let us consider the supergraphs given by Figs.2a – 2c in more details. The contribution from the supergraph given at Fig.2a is equal to (see (6))

$$\begin{aligned} I_a &= \int d^8\theta_1 d^8\theta_2 \int du_1 du_2 du_3 dw_1 dw_2 dw_3 \frac{1}{\widehat{\Box}} \delta^8_{12} (\mathcal{D}^+_2)^4 (\mathcal{D}^+_3)^4 \delta^8_{12} V^{++}(1) V^{++}(2) \times \\ &\times \frac{1}{\widehat{\Box}} \frac{1}{(u^+_1 u^+_2)(u^+_2 u^+_3)(u^+_1 u^+_3)(w^+_1 w^+_2)(w^+_2 w^+_3)(w^+_1 w^+_3)} \times \\ &\times \delta^{(-2,2)}(u_2, w_2) \delta^{(-2,2)}(u_3, w_3) \end{aligned} \tag{12}$$

We take into account that $\mathcal{D}^+$ commutes with $\widehat{\Box}$ and use the relation $\delta^8_{12} (\mathcal{D}^+_2)^4 (\mathcal{D}^+_3)^4 \delta^8_{12} = (u^+_2 u^+_3)^4 \delta^8_{12}$. Integration over w_2 and w_3 leads to

$$I_a = \int d^8\theta \int du_1 du_2 du_3 dw_1 \frac{1}{\widehat{\Box}^2} V^{++}(1) V^{++}(2) \frac{(u^+_2 u^+_3)^2}{(u^+_1 u^+_2)(u^+_1 u^+_3)(w^+_1 u^+_2)(w^+_1 u^+_3)} \tag{13}$$

This expression coincides with the result obtained in [18], the only difference consists in presence of $\widehat{\Box}$ instead of $\Box$. However since superfield strength $\mathcal{W}$ does not depend on harmonic coordinates we can carry out the trick which was used in [18]. We express $(u^+_2 u^+_3)^2$ as $D^{++}_2 D^{++}_3 [(u^-_2 u^-_3)(u^+_2 u^+_3)]$ and integrate by parts to transfer $D^{++}_2 D^{++}_3$ to other terms of I_a. After that the I_a takes the form

$$I_a = \int d^8\theta \int du_1 dw_1 \frac{1}{\widehat{\Box}^2} V^{++}(1) V^{++}(2) \frac{u^+_1 w^-_1}{u^+_1 w^+_1} \tag{14}$$

Analogous consideration allows to show that contributions from supergraphs given by Fig.2b, Fig.2c are respectively equal to

$$I_b = -2 \int d^8\theta \int du_1 dw_1 \frac{1}{\widehat{\Box}^2} V^{++}(1) V^{++}(2) \frac{(u^+_1 w^-_1)(u^-_1 w^+_1)}{(u^+_1 w^+_1)^2} \tag{15}$$

and

$$I_c = -2 \int d^8\theta \int du_1 dw_1 \frac{1}{\widehat{\Box}^2} V^{++}(1) V^{++}(2) \frac{1}{(u^+_1 w^+_1)^2} \tag{16}$$

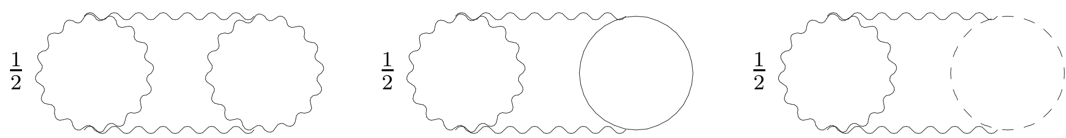
Here $V^{++}(1)$, $V^{++}(2)$ are external gauge lines (which are contracted to some systems of propagators or vertices in multiloop supergraphs containing graphs given at Fig.2a – Fig.2c as subdiagrams). These contributions are analogous to corresponding expressions given in [18] with the only difference in presence of $\widehat{\Box}$ instead of $\Box$. It is evident that $I_1 + I_2 + I_3 = 0$ in accordance with [18], i.e. these supergraphs cancel each other in sector of constant background fields. This effect provides vanishing of sum of supergraphs given by Fig. 3a – Fig. 3c, Fig. 4a – Fig. 4c and Fig. 5.

We note that vanishing of this sum is caused by $N = 4$ supersymmetry and does not depend on structure of (Abelian) gauge group. Therefore such a situation is common for any

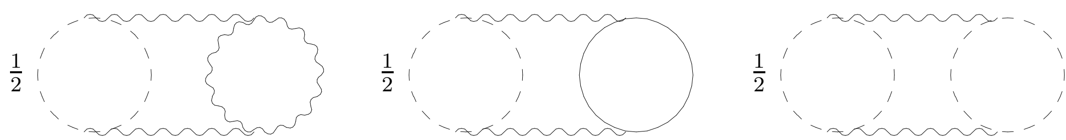
$SU(n)$ gauge group broken down to its maximal Abelian subgroup independently of value of n.

Let us return back to the supergraphs given by Fig. 1a. Each supergraph has some combinatoric factor. The straightforward calculations show that they are proportional to each other with the coefficients 1/2 or 1 which are written near the corresponding supergraphs. We call these coefficients the relative factors.

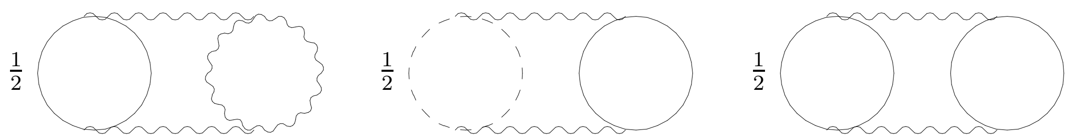
The supergraphs given by Fig. 1a can be equivalently regroupped into the sets of $N = 4$ superpartners as shown on Figs. 3a - 3c where the relative factors are also written near the supergraphs. All supergraphs on Fig. 3a are $N = 4$ superpartners as well as the supergraphs on Figs. 3b, 3c respectivelly. As a result one gets the three sets of the $N = 4$ superpartner supergraphs. Each such a set can be studied by the same methos as the supergraphs on Figs. 2a - 2c. It leads to conclusion that sum of the contributions of the supergraphs in every set is equal to zero.



$Fig.3a$



$Fig.3b$



$Fig.3c$

Now we turn to remaining three-loop supergraphs including six propagators which are given by Fig. 1b. There are also two pairs of supergraphs analogous to this pair, they include ghost and gauge propagators instead of matter propagators. After D-algebra transformations contributions of both these supergraphs and their analogs in which matter loop is replaced by gauge and ghost loop respectively turn to be proportional to the following integral over internal momenta:

$$\begin{aligned} J &= tr \int d^8\theta \int \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^{12}} \frac{1}{(k_1^2 + \mathcal{W}\bar{\mathcal{W}})(k_2^2 + \mathcal{W}\bar{\mathcal{W}})(k_3^2 + \mathcal{W}\bar{\mathcal{W}})} \times \\ &\times \frac{1}{((k_1+k_2)^2 + \mathcal{W}\bar{\mathcal{W}})((k_1+k_3)^2 + \mathcal{W}\bar{\mathcal{W}})((k_2+k_3)^2 + \mathcal{W}\bar{\mathcal{W}})} \end{aligned} \qquad (17)$$

Here tr is a matrix trace. We use the expression for $N = 2$ background superfield strength in the form (8), the operator $\widehat{\Box}$ in the form (9). The integral in the expression (17) is formally logarithmically divergent (although supergraphs without matter legs and legs including derivatives of gauge strengths have superficial degree of divergence equal to zero

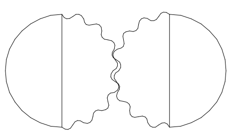
[12], the corresponding divergences vanish due to supersymmetry). Therefore we carry out dimensional regularization via changing integration over d^4k_i by integration over $d^{4+\epsilon}k_i$ (with $i = 1, 2, 3$). Straightforward calculation of J leads to the result

$$J = tr\frac{1}{(16\pi^2)^3}\int d^8\theta(\frac{2}{\epsilon} + \log(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2})) \tag{18}$$

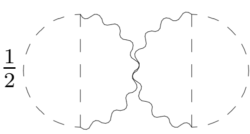
However pole part vanishes due to known properties of integration over anticommuting variables (see f.e. [21]). Since $\mathcal{W}, \bar{\mathcal{W}}$ are the diagonal matrices they commute with each other, and we can use identity $\int d^8\theta \log(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2}) = \int d^8\theta(\log(\frac{\mathcal{W}}{\mu}) + \log(\frac{\bar{\mathcal{W}}}{\mu}))$. This expression vanishes due to chirality of $\mathcal{W}$. Therefore the supergraphs given by Fig. 1b give zero contribution. The same situation takes place for analogous supergraphs where the matter propagators are replaced by gauge and ghost ones. Hence we conclude that three-loop contribution to non-holomorphic effective potential is equal to zero.

Now let us consider four-loop supergraphs. It turns to be that the situation we observed at three-loop order takes place also at four-loop order, i.e. each supergraph either has $N = 4$ superpartners sum together with which it is equal to zero or gives the contribution proportional to $\int d^{12}z \log(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2}) = 0$.

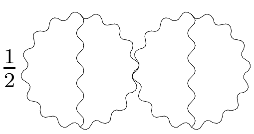
First of all, at four-loop order we get two systems of supergraphs given by Figs. 4.1 – 4.6 and Fig.5 which can be separated into sets of $N = 4$ superpartners. The straightforward calculations show that the combinatoric factors of different supergraphs at Figs. 4.1-4.6 are proportional to each other with the relative factors 1 or 1/2. These factors are written near of each supergraph.

$\frac{1}{2}$ 

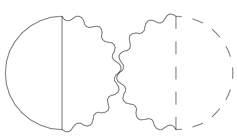
$Fig.4.1$

$\frac{1}{2}$ 

$Fig.4.2$

$\frac{1}{2}$ 

$Fig.4.3$

1 

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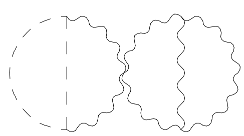
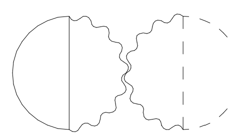
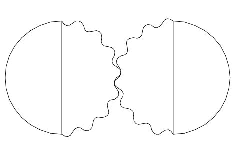
1 

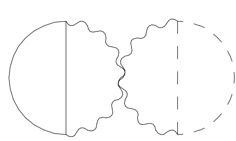
$Fig.4.5$

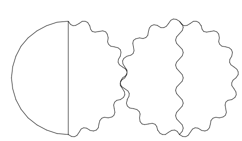
1 

$Fig.4.6$

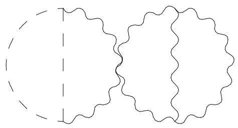
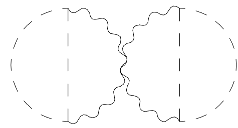
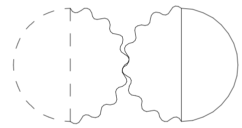
The scheme of separation of supergraphs given by Fig. 4.1 – Fig. 4.6 into sets of $N = 4$ superpartners is given by Fig. 4a – Fig. 4c where the relative factor is also manifestly shown near corresponding supergraph. We again get three sets of the $N = 4$ superpartner supergraphs as in three-loop case (see Figs. 3a - 3c). Each line on the Figs. 4a – 4c contains the superpartner supergraphs.

$\frac{1}{2}$ 

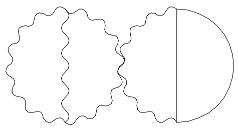
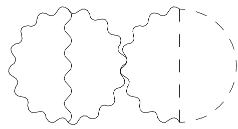
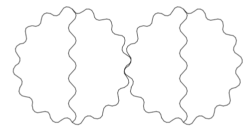
$\frac{1}{2}$ 

$\frac{1}{2}$ 

$Fig.4a$

$\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$ 

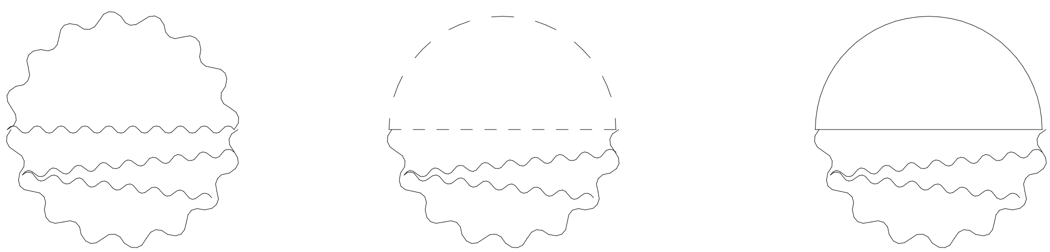
$Fig.4b$

$\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$ 

$Fig.4c$

It is evident that the total contribution of the supergraphs given by Figs. 4a – 4c is equal to that one of the supergraphs given by Figs. 4.1 – 4.6. Since the supergraphs within every set are the superpartners their total contribution vanishes. It is proved by the same method as three loop order.

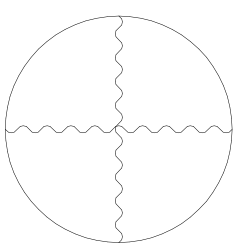
Another system of $N = 4$ superpartner supergraphs at four-loop order is given by Fig.5.



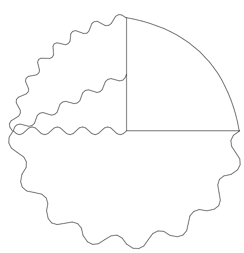
$Fig.5$

Since these supergraphs are $N = 4$ superpartners, it is easy to see that sum of contributions of these supergraphs is equal to zero.

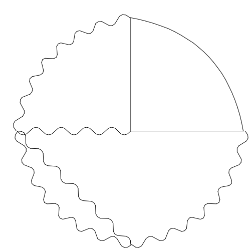
The remaining four-loop supergraphs with eight propagators are given by Figs. 6a – 6c.



$Fig.6a$



$Fig.6b$



$Fig.6c$

The contributions of these supergraphs after D-algebra transformations are proportional to the following integrals over internal momenta respectively:

$$\begin{aligned} J_{6a} &= tr \int d^8\theta \int \frac{d^4k_1 d^4k_2 d^4k_3 d^4k_4}{(2\pi)^{16}} \frac{1}{(k_1^2 + \mathcal{W}\bar{\mathcal{W}})(k_2^2 + W\bar{W})(k_3^2 + \mathcal{W}\bar{\mathcal{W}})} \times \\ &\times \frac{1}{((k_1+k_2)^2 + \mathcal{W}\bar{\mathcal{W}})((k_1+k_2+k_3)^2 + \mathcal{W}\bar{\mathcal{W}})((k_1+k_4)^2 + \mathcal{W}\bar{\mathcal{W}})} \times \end{aligned}$$

$$
\begin{aligned}
&\times \quad \frac{1}{(k_4^2+\mathcal{W}\bar{\mathcal{W}})((k_2+k_3-k_4)^2+\mathcal{W}\bar{\mathcal{W}})} \\
J_{6b} &= tr\int d^8\theta \int \frac{d^4k_1 d^4k_2 d^4k_3 d^4k_4}{(2\pi)^{16}} \frac{1}{(k_1^2+\mathcal{W}\bar{\mathcal{W}})(k_2^2+\mathcal{W}\bar{\mathcal{W}})(k_3^2+\mathcal{W}\bar{\mathcal{W}})} \times \\
&\times \quad \frac{1}{((k_1+k_2)^2+\mathcal{W}\bar{\mathcal{W}})((k_1+k_3)^2+\mathcal{W}\bar{\mathcal{W}})((k_3+k_4)^2+\mathcal{W}\bar{\mathcal{W}})} \times \\
&\times \quad \frac{1}{(k_4^2+W\bar{W})((k_1+k_3+k_4)^2+\mathcal{W}\bar{\mathcal{W}})} \\
J_{6c} &= tr\int d^8\theta \int \frac{d^4k_1 d^4k_2 d^4k_3 d^4k_4}{(2\pi)^{16}} \frac{1}{(k_1^2+\mathcal{W}\bar{\mathcal{W}})(k_2^2+\mathcal{W}\bar{\mathcal{W}})(k_3^2+\mathcal{W}\bar{\mathcal{W}})} \times \\
&\times \quad \frac{1}{((k_1+k_2)^2+\mathcal{W}\bar{\mathcal{W}})((k_2+k_3)^2+\mathcal{W}\bar{\mathcal{W}})((k_2+k_3-k_4)^2+\mathcal{W}\bar{\mathcal{W}})} \times \\
&\times \quad \frac{1}{(k_4^2+\mathcal{W}\bar{\mathcal{W}})((k_1+k_2+k_4)^2+\mathcal{W}\bar{\mathcal{W}})} \qquad (19)
\end{aligned}
$$

After dimensional regularization and integration the J_{6a}, J_{6b}, J_{6c} turn to be equal to

$$
J_{6a} = J_{6b} = J_{6c} = tr\frac{1}{(16\pi^2)^4}\int d^8\theta\left(\frac{2}{\epsilon} + \log\left(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2}\right)\right) \qquad (20)
$$

This expression is completely analogous to (18). As a result we get the same conclusion. Each of supergraphs given by Figs. 6a – 6c is proportional to $\int d^{12}z \log(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2})$ where $\mathcal{W}$ is diagonal matrix of the form (8) and $\bar{\mathcal{W}}$ is its conjugate. The same situation takes place for the analogous supergraphs where the matter superpropagators are replaced by gauge and ghost ones. Hence their contributions also vanish. By the way, we convinced that terms of the form $\frac{\mathcal{W}_a-\mathcal{W}_b}{\mathcal{W}_c-\mathcal{W}_d}$ supposed in [8,15] do not arise in non-holomorphic effective potential at least at three and four loops.

We investigated all four-loop supergraphs including eight internal lines. Namely for this number of superpropagators all D-factors are used for contracting loops to points in θ-space. The four-loop supergraphs with nine superpropagators are also present but after D-algebra transformations in such supergraphs the extra factor $(\mathcal{D}^+)^4$ remains. It can act only on background superfield strengths. Therefore four-loop supergraphs with nine propagators cannot contribute to non-holomorphic effective potential.

To conclude, we have considered three- and four-loop supergarphs contributing to non-holomorphic effective potential and proved that only two situations are possible for these supergraphs: (i) either contribution of this supergraph is proportional to $\int d^{12}z \log(\frac{\mathcal{W}\bar{\mathcal{W}}}{\mu^2})$, and such a structure vanishes due to properties of integral in superspace (ii) or such a supergraph has $N=4$ superpartner supergraphs, and sum of contribution from three $N=4$ superpartner supergraphs is equal to zero because of $N=4$ supersymmetry. This result is common for any unitary gauge group broken down to its maximally symmetric torus since vanishing of sum of superpartner supergraphs is caused only by $N=4$ supersymmetry, not by structure of gauge group.

We found that the mechanism of vanishing of corrections to non-holomorphic effective potential at three-loop order essentially differs from that one at two loops. Absence of two-loop contributions is stipulated only by $N = 2$ supersymmetry. $N = 4$ supersymmetry begins to work efficiently at three loops and higher and manifests itself by means of $N = 4$ superpartner supergraphs. However we proved that the situation at four loops is completely analogous to one in the previous order. The mechanism of vanishing the three- and four-loop contributions to non-holomorphic effective potential looks like very generic and one can expect that it works at any loop.

Detailed study of the structure of above three- and four- loop supergraphs and explicit results of their calculations will be published elsewhere.

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